Chapter One

Introduction to Digital Signal Processing

1-1 Definitions

A **signal** is a piece of information (natural or synthetic) explained as a function of time (and perhaps other variables like the dimensions x, y, etc.). This information can be represented by the variations in the signal amplitude, phase, or frequency.

A **system** is a physical or mathematical (i.e., hardware or software) entity that performs operations on signals to extract or modify information. For example, a low-pass filter is a system that removes high frequency content from the signal.

Digital signal processing (DSP) is the use of digital processing by computers or more specialized digital signal processors, to perform a wide variety of signal processing operations. The signals processed in this manner are a sequence of numbers that represent samples of a continuous variable in a domain such as time, space, or frequency.

1-2 Applications of DSP

Applications of DSP include audio signal processing, audio compression, digital image processing, video compression, speech processing, speech recognition, digital communications, digital synthesizers, radar, sonar, financial signal processing, seismology and biomedicine. Specific examples include speech coding and transmission in digital mobile phones, room correction of sound in hi-fi and sound reinforcement applications, weather forecasting, economic forecasting, seismic data processing, analysis and control of industrial processes, medical imaging such as CAT scans and MRI, MP3 compression, computer graphics, image manipulation, audio crossovers and equalization, and audio effects units. Figure 1.1 shows DSP applications in engineering.

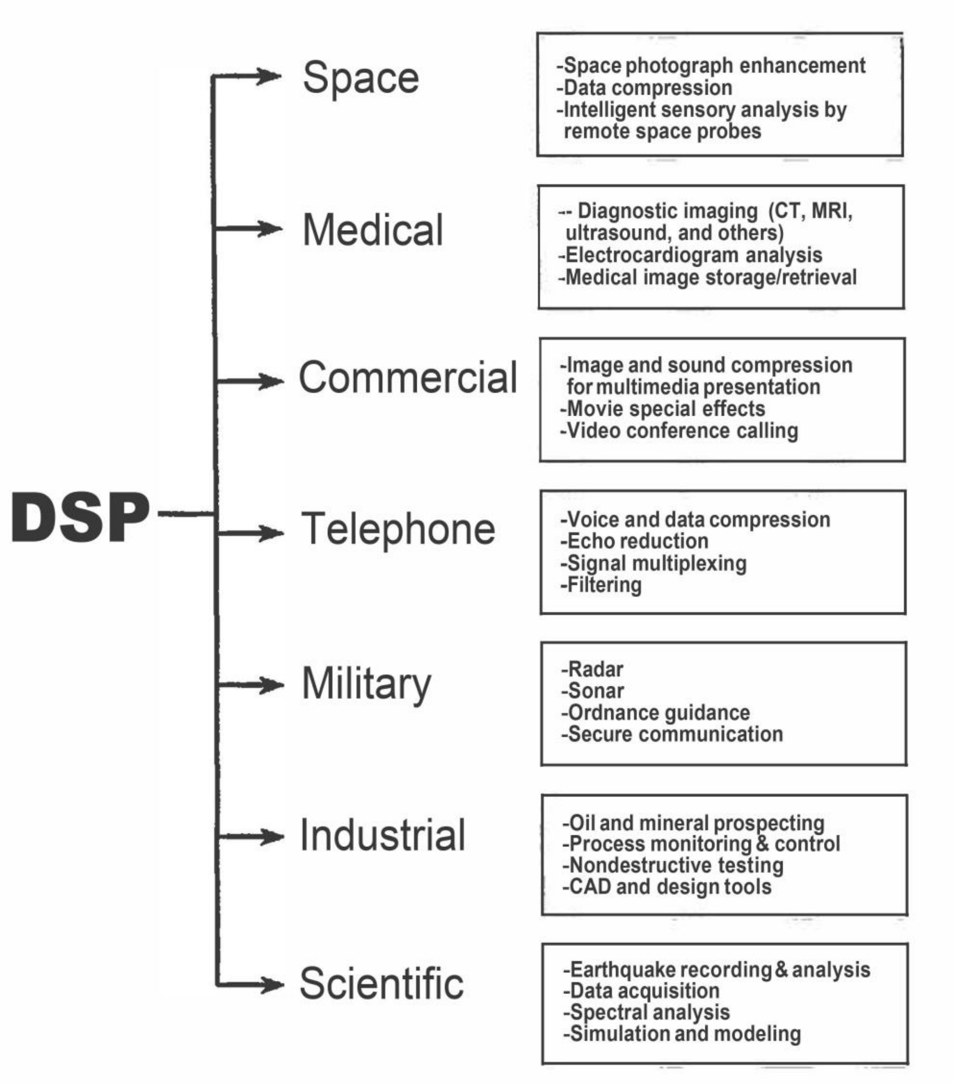

Figure 1 : Engineering applications of DSP

1-3 Classification of Signals

1-3.1 Analog, discrete, and digital:

Analog: defined as continuous-time representation.

Discrete: defined only at discrete time instants.

Digital: discrete, quantized (to specific levels), and constant between adjacent discrete instants [Figure (2)]. Analog signals can be processed only by physical systems (hardware), while digital signals can be processed by hardware or software systems

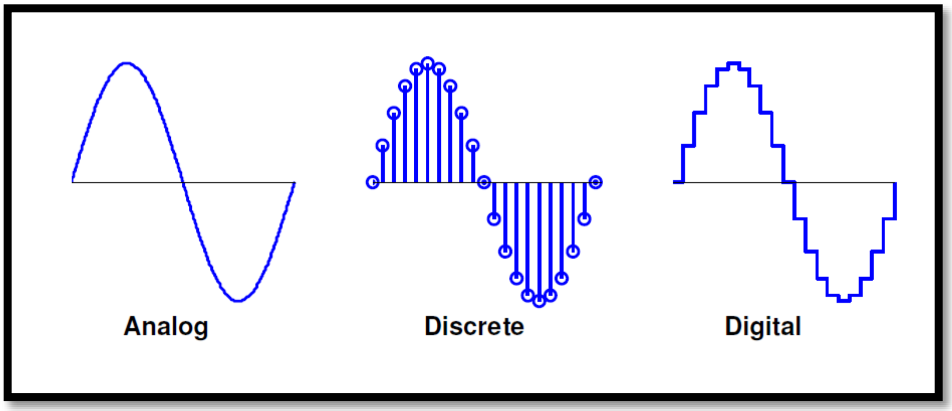

Figure 2: Analog, Discrete and Digital Signals.

1-3.2 Periodic & non-periodic:

A signal which repeats itself after a fixed time period or interval is called as **periodic** signal, and is represented by x (t) = x (t + T_0). This is called as condition of periodicity. Here T_0 is called as fundamental period. That means after this period the signal repeats itself. For the discrete time signal, the equation becomes: x (n) = x (n + N). Here number 'N' is the period of signal. Periodic signals are shown in Figure (3), a and b.

A signal which does not repeat itself after a fixed time period or does not repeat at all is called as **non-periodic** or **aperiodic** signal. Thus x (t) is not equal to x (t + T_0). And x (n) is also not equal to x (n + N). Non-periodic signal is shown in Figure (3) c.

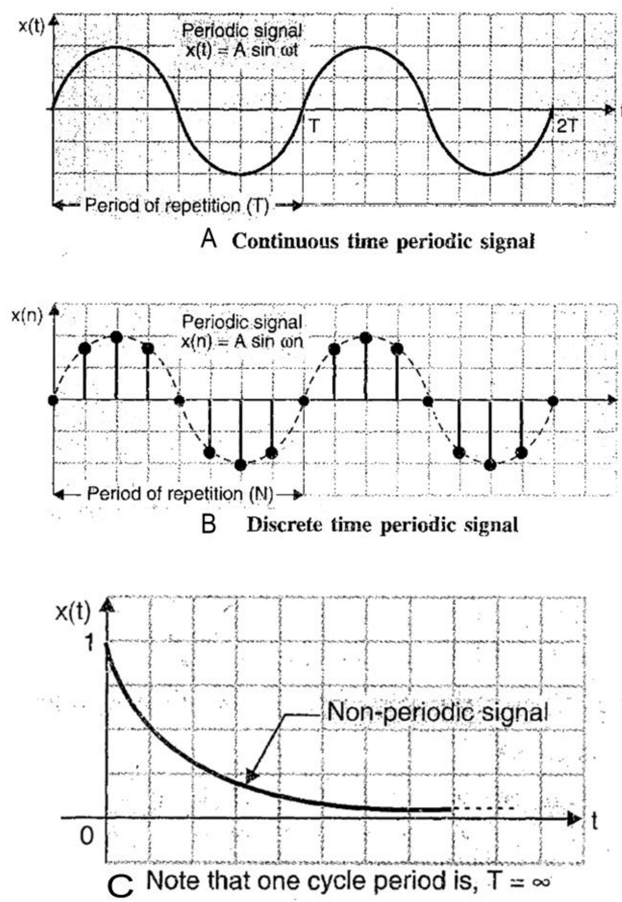

Figure (3): (A) continuous sine, (B) discrete sine, (C) exponential, signals.

1-3.3 Multi-channel, Multi-Dimensional signals:

A Multi-channel signal is expressed by: x (t) = (x_1 (t), x_2 (t), x_3 (t) … x_N (t),)

A Multi- Dimensional signal is expressed by: x (t_1, t_2, t_3 … t_N)

1-3.4 Mono-component and Multi-component:

This depends on how many distinct frequencies exist in the signal. Figure (4) shows a digitized audio signal and its calculated signal spectrum (frequency content). The first signal is mono and the one below is multi component.

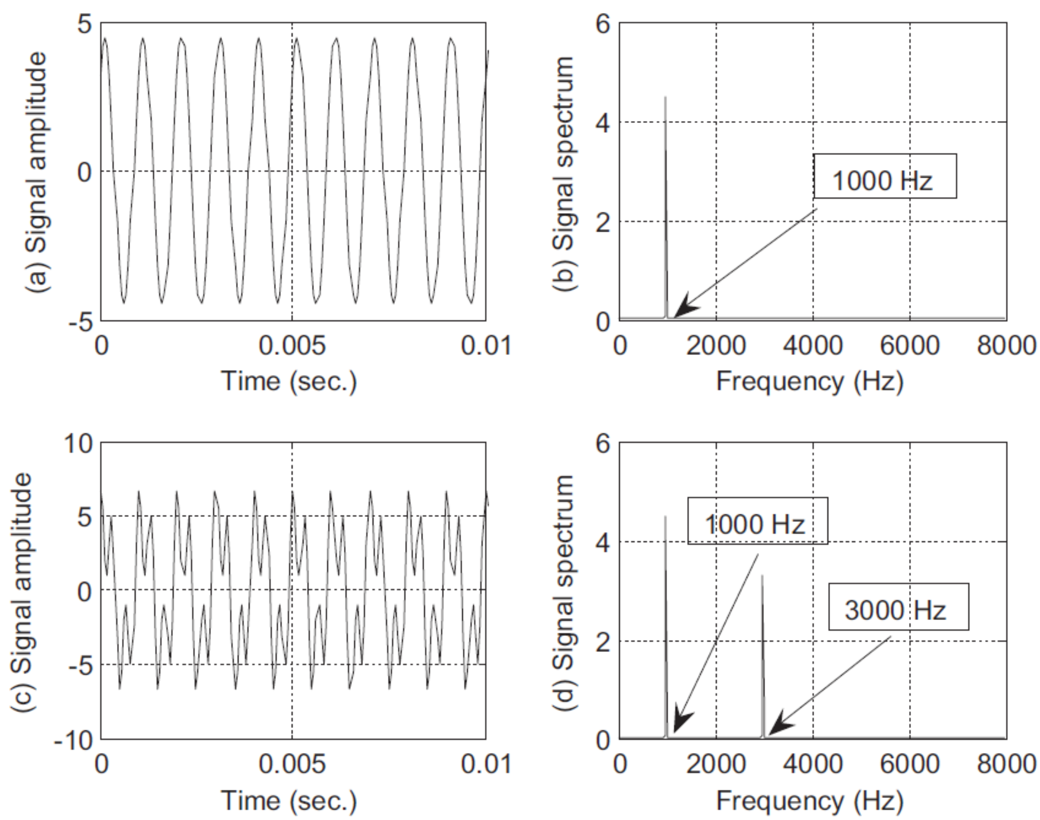

Figure (4): Mono-component and Multi-component signals, and their spectrum.

1-3.5 Deterministic and random:

For example, x (t) = sin (w_0 t) is deterministic, i.e., its exact value is known at any time, while noise n(t) is random (cannot be determined exactly as a function of time).

1-4 Classification of Systems

1-4.1 Analog (i.e. continuous-time), discrete, and digital systems

1-4.2 Time-varying and time-invariant:

If the input x(t) , which gives an output y(t) , is shifted in time by t_0 , i.e., the new input is $x(t - t_0)$, then a time-invariant system will give an output which is the same as y(t) but time-shifted by the same amount to , i.e., y(t − to), as illustrated in Figure (5).

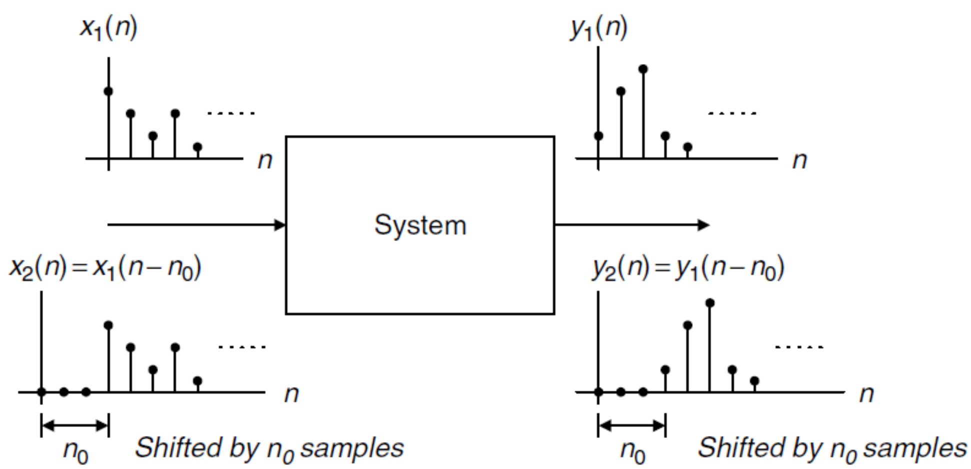

Figure (5): Illustration of time- invariant system.

1-4.3 Causal and non-causal systems:

The output of a causal system is not dependent on future values of the input signal x (t). i.e. y (t) is not a function of $x(t + t_0)$.

1-4.4 Static (memoryless) and dynamic (with memory) systems:

A system whose output does not depend on a previous value of the input signal x (t) is called memoryless, i.e., y (t) is not a function of $x(t - t_0)$.

1-4.5 Linear and non-linear systems:

A system T is called homogeneous if it satisfies the scaling property:

$$T[c \times x(t)] = cT[x(t)] \text{ , where c is a constant,}$$

and is called additive if it satisfies the additivity condition:

$$T[\, x_1(t) + x_2(t)\,] = T[x_1(t)] + T[x_2(t)\,].$$

A **linear** system satisfies both of the above conditions.

1-5 Basic components of a DSP system

Most signals in nature or in communication systems are analog. To process those signals using digital systems, analog-to-digital (A/D) conversion is necessary. After processing, digital-to-analog (D/A) conversion is applied to obtain the modified analog signal. The general signal processing system is shown in Figure (6).

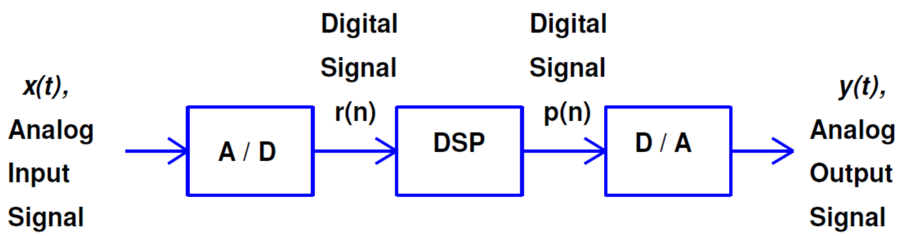

Figure (6): Basic components of a DSP system.

An analog to digital convertor (A/D) consists of a Sampler and a Quantizer. Figure (7) shows the (A/D) conversion process.

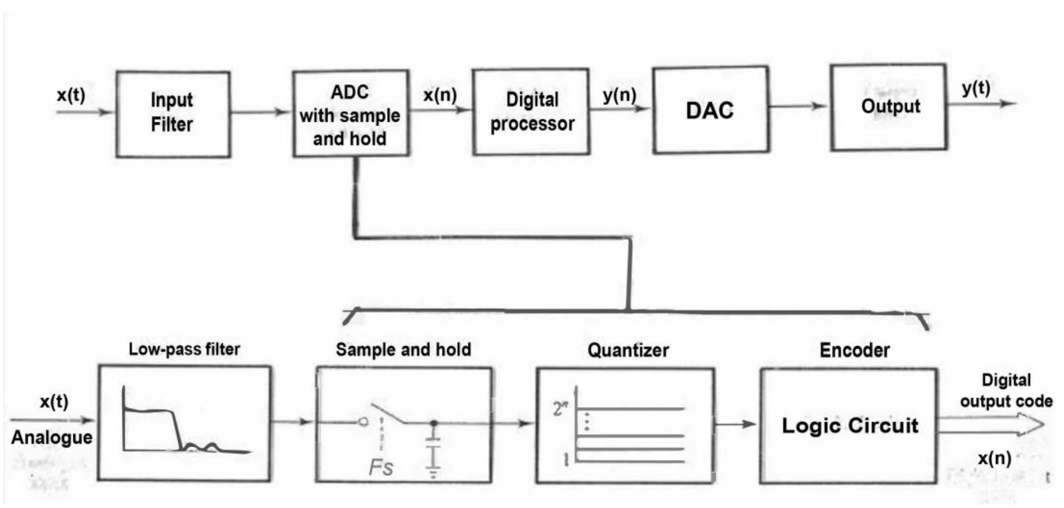

Figure (7): (A/D) conversion process.

1-6 Advantages of Digital over Analog Signal processing (DSP Vs ASP)

- DSP is less susceptible to noise and power supply disturbances than ASP.
- DSP is more accurate, especially in reading the results of signal processing.
- Storage of digital signals is easier.
- DSP is more flexible and versatile, especially in changing the system parameters to handle changing environments (e.g., in adaptive filtering).

1-7 Sampling Theorem

Generally, sampling is considered as any process that records a signal at discrete instances. In uniform sampling, samples are equally spaced from one another by a fixed sampling interval T. The reciprocal of the sampling interval is called the sampling frequency (or sampling rate) $F_S = 1 / T$ which has units of hertz. Lossless sampling requires **$F_S > 2B$,** where B is the bandwidth of the signal (i.e. width of the spectrum). This condition is called the Nyquist sampling theorem that is illustrated in Figure (8) which is:

- Part (a) is the original continuous signal
- Part (b) is a train of pulses.
- Part (c) is the discrete signal resulted from the multiplication of signals of parts (a) and (b)
- Part (d) is the spectrum (Fourier transform) of the original signal (part (a))
- Part (e) is the spectrum of part (b)
- Part (f) is the spectrum of the discrete signal of part (c).

As seen in Figure (8), if F_S is less than 2B, Part (e) components will overlap, and the original signal will be distorted.

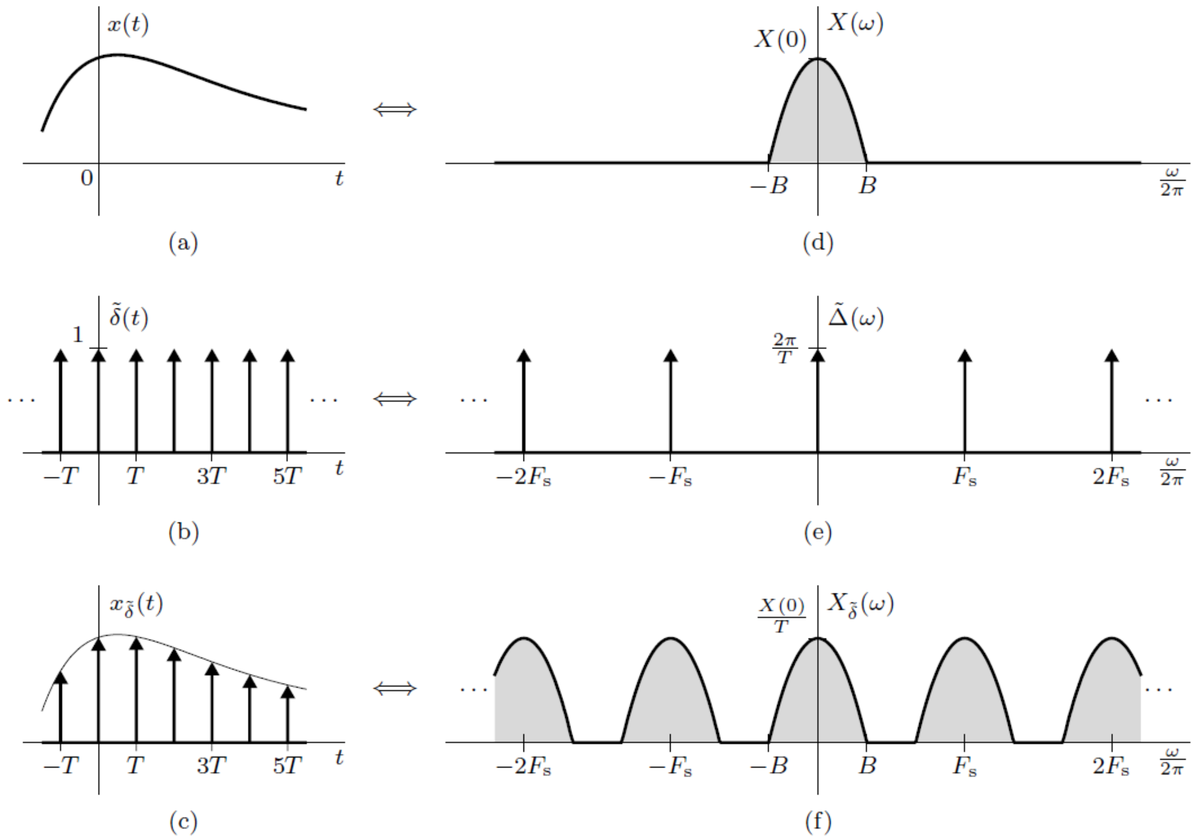

Figure (7): Sampling Theorem.

1-8 Examples and Tutorial

1-8.1 Example 1 : Interference Cancellation in Electrocardiography (ECG)

In ECG recording, there often is unwanted 60-Hz interference in the recorded data. Using proper grounding or twisted pairs minimizes such 60-Hz effects, another effective choice can be used here is a digital notch filter, which eliminates the 60-Hz interference while keeping all the other useful information. Figure (8) shows a DSP example.

1-8.2 Example 2 : Digital Image Processing: Image Restoration

Image restoration refers to that process by which a blurred image is restored to its original focused condition. Figure (9) illustrates this process.

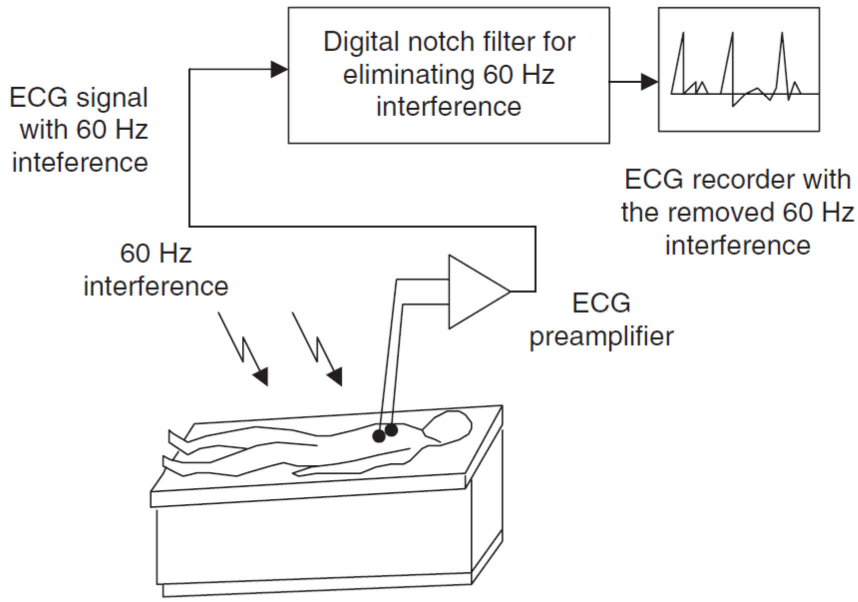

Figure (8): Elimination of 60-Hz interference in electrocardiography (ECG).

Figure (9): An example of image de-blurring. (a) Original blurred image. (b) Restored image

1-8.3 Example 3: the system [y (t) = x (t) + 2] is not linear. This system can be represented by the operator T such that T [x(t)] = x(t) + 2. Assume that $x(t) = a \times x_1(t) + b \times x_2(t)$, where a and b are constants. Now we have:

$$T[a \cdot x_1(t) + b \cdot x_2(t)] = T[x(t)] = x(t) + 2 = a \cdot x_1(t) + b \cdot x_2(t) + 2$$
$$\neq aT[x_1(t)] + bT[x_2(t)] = a[x_1(t) + 2] + b[x_2(t) + 2] = ax_1(t) + bx_2(t) + 2a + 2b\,.$$

Hence, a system with independent internal sources is not linear.

1-8.4 Example 4: The system y (t) = ln [x(t)] is non-linear since ln [c * x(t)] is not equal to c * ln [x(t)] .

1-8.5 Example 5: A discrete-time system is defined by the following difference equation:

$$y[n] = x[n+1] - 2x[n] + x[n-1]$$

Is it (a) time-invariant? (b) Is it causal?

Solution:

(a) From the given difference equation, we notice that delaying the input sequence by an integer makes the system time-invariant.
(b) It is noticed from the given system's input-output relationship that the response of the system at the current time index n depends on the input at the next future input {n+1}. Therefore, the system is anticipatory and hence is non-causal.

1-8.6 **Example 6:** Is the system defined by the equation below causal

$$y[n] = K + x[n] + 0.75x[n-1],$$

Solution:

The system response at the current time index n does not depend on the input sequence at future time instants. Hence the system is causal

1-8.7 **Example 7:** Find the minimum sampling frequency (F_S) and max time interval (max Sampling time T_S) if the Signal maximum frequency (Fm or B) = 3 kHz.

Solution:

F_S = 2B = 2 * 3 kHz = 6 kHz, this is the minimum sampling frequency.

T_S max = 1 / F_S = 1 / 6 kHz = 0.166 ms (Milliseconds) this is the max time interval.

1-9 References

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